Exercise 1 (True and Safe Updates). Consider the following set of Datalog rules:

\[
\begin{align*}
  p(X) &\leftarrow r(X), \text{not } d(X). \\
  p(X) &\leftarrow q(X, Y, Z), s(X), Y > X + A, t(\_ , A).
\end{align*}
\]

a) Determine all propagation rules which are necessary in order to compute true induced insertions \( p^+ \) and deletions \( p^- \) with respect to relation \( p \).

b) Determine all propagation rules which are necessary in order to compute safe induced deletions \( p^- \) with respect to relation \( p \) using the smallest set of side literals possible.

Exercise 2 (Mixing Propagation Rules). Reconsider the the rules from Exercise 1 and assume that the relations \( r \) and \( q \) are derived ones.

a) Determine all propagation rules for propagating true insertions \( p^+ \) assuming that for the relations \( r \) and \( q \) solely safe updates are provided in \( r^+ \) and \( q^+ \).

b) Determine all propagation rules for propagating true insertions \( p^+ \) assuming that for the relations \( r \) and \( q \) solely potential updates are provided in \( r^+_b \) and \( q^+_{f/b} \).

c) Determine all propagation rules for propagating safe insertions \( p^+ \) assuming that for the relations \( r \) and \( q \) solely potential updates are provided in \( r^+_b \) and \( q^+_{f/b} \).

Exercise 3 (Potential Updates). Consider the following set of Datalog rules (\( b_i \) assumed to be base relations):

\[
\begin{align*}
  p(X) &\leftarrow q(X, Z), r(Z), X > Z, s(Z, Y). \\
  r(Z) &\leftarrow b_1(Z, \_ , Z, d(Z, X)). \\
  d(Z, X) &\leftarrow b_2(Z, D), b_3(D, X)
\end{align*}
\]

a) Determine two different but complete sets of propagation rules for computing potential insertions \( p^+_b \) with respect to relation \( p \).

b) How many different sets of propagation rules for potential updates can be derived per rule set?