Exercise 1 (Magic Sets). Consider the following rules and facts:

Rules R:

\[
\begin{align*}
    p(X,Y) & \leftarrow e(X,Y). \\
p(X,Y) & \leftarrow e(X,Z), p(Z,Y).
\end{align*}
\]

Facts F:

\[
\begin{align*}
e(8,1), e(9,1), e(1,2), e(1,4), e(2,3), e(5,6), e(6,7).
\end{align*}
\]

a) Determine the Magic Sets transformed rules for R and the query \( p(1,7) \) using the SIPS left-to-right and right-to-left.

b) Compute the fixpoint \( F^* \) of \((R,F)\) and compare the intermediate results with the facts generated for the Magic Sets transformed rules.

c) Identify the different search strategies realized by the transformed rule sets by visualizing the search graphically.

d) Determine the Magic Sets transformed rules for query \( p(X,7) \) using the SIPS left-to-right and show the benefits of employing a partial SIPS rather than a full one.

Exercise 2 (Properties of Magic Sets). Consider the following rules:

\[
\begin{align*}
p(X) & \leftarrow b(X,Y,Z), \neg q(Y), \neg q(Z). \\
q(X) & \leftarrow b(X, \ldots).
\end{align*}
\]

a) Determine the Magic Sets transformed rule set for the query \( p(1) \). Is the resulting rule set still stratifiable?

b) Compute the fixpoint for the following facts: \( b(1,1,2), b(1,2,2), b(2,1,2), b(2,2,2), b(3,4,5) \).