The following proof exemplarily illustrates how the correctness of rule transformations can be shown.

**Proposition:**
Let $D^{\text{old}}=(F^{\text{old}},R)$ be a stratifiable database, $\delta$ a transition and $\Delta=\langle \Delta^+,\Delta^- \rangle$ the corresponding induced transition from $D^{\text{old}}$ to $D^{\text{new}}$. Let $D^a=(\delta(F),R \cup R^\Delta)$ be the augmented database of $D$. Then the delta relations defined by the propagation rules $R^\Delta$ correctly represent the induced transition $\Delta$. Hence, for each relation $p \in \text{pred}(D)$ the following conditions hold:

\[
\begin{align*}
p^+(\bar{t}) \in F^*_D & \iff p(\bar{t}) \in \Delta^+ \\
p^-(\bar{t}) \in F^*_D & \iff p(\bar{t}) \in \Delta^-
\end{align*}
\]

$\Rightarrow$ Proof by induction on the depth of proof trees for a literal $A$
Example of a proof tree:

\[
\begin{align*}
R: & \quad s(X,Y) \leftarrow p(X,Y). \\
& \quad p(X,Y) \leftarrow p(X,Z), r(Z,a,Y). \\
& \quad p(X,Y) \leftarrow p(X,Z), r(Z,a,W), p(W,Y). \\
& \quad p(X,Y) \leftarrow r(X,b,Y).
\end{align*}
\]

\[
\begin{align*}
F: & \quad r(1,b,2), r(2,a,3), r(3,a,4), r(4,b,5).
\end{align*}
\]
Correctness of Propagation Rules - 3

Showing that

\[ p^+(\overline{t}) \in F^*_{D^a} \iff p(\overline{t}) \in \Delta^+ \]

**Proof:**
The proposition is shown by induction on the depth of proof trees for \( A \) (resp. \( A^+ \)) with respect to \( D^\text{new} \) (respectively \( D^a \)). We assume that the meta predicates \( \text{old} \) and \( \text{new} \) are correctly evaluated with respect to the database states \( F^*_{\text{old}} \) and \( F^*_{\text{new}} \), respectively.

**Case:**

\[ p^+(\overline{t}) \in F^*_{D^a} \iff p(\overline{t}) \in \Delta^+ \]

Induction on the depth \( d \) of proof trees with respect to \( D^\text{new} \) that the implication \( A \in \Delta^+ \Rightarrow A^+ \in F^*_{D^a} \) holds. Suppose that \( A \in \Delta^+ \)

\[ \Rightarrow \exists \text{ a proof tree for } A \text{ with respect to } D^\text{new} \text{, but none with respect to } D^\text{old}. \]
In this case, $A$ refers to an extensional relation and hence $A^+ \in \delta(F) \subseteq F^*_{D^a}$.

We assume that the implication holds for all atoms in $\Delta^+$ having a proof tree with respect to $D^{new}$ of depth less than $d$. As $d > 0$, $A$ has children $L_1\sigma, \ldots, L_n\sigma$ and $A \equiv B\sigma$ is derived via

$$R \equiv B \leftarrow L_1, \ldots, L_n$$

As no proof tree exists for $A$ with respect to $D^{old}$, at least one of the $L_i\sigma$ is not derivable in $D^{old}$.

If $L_i\sigma$ is a positive literal, then $L_i\sigma \in \Delta^+$. As $L_i\sigma$ has a proof tree of depth $< d$ with respect to $D^{new}$, $L^+_i\sigma \in F^*_{D^a}$ follows from the induction hypothesis.

If $L_i\sigma \equiv \neg C_i\sigma$ is a negative literal, then $C_i\sigma \in \Delta^-$. It can be shown by induction on the depth of proof trees with respect to $D$ that then $L^+_i\sigma \equiv C^-_i\sigma \in F^*_{D^a}$ holds, but this part of the proof is omitted since it can be performed by analogy to the proof for deletions.
Correctness of Propagation Rules - 5

From these two cases follows that \( L_i^+ \sigma \in F_{D^a}^* \). Since for each body literal of \( R \) a positive propagation rule is generated such that the rule

\[
B^+ \leftarrow L_{i+1}^{+ \text{new}}, \ldots, L_{i+1}^{\text{new}}, L_{i+1}^{\text{new}}, \ldots, L_n^{\text{new}}, \text{old not } B.
\]

is in \( R^A \). As evaluations over new and old are correct, it follows that \( A^+ \equiv B^+ \sigma \) is derivable in \( D^a \), i.e., \( A^+ \in F_{D^a}^* \).

Case:

\[
p^+(\vec{t}) \in F_{D^a}^* \quad \Rightarrow \quad \bar{p}(\vec{t}) \in \Delta^+
\]

Induction on the depth \( d \) of proof trees with respect to \( D^a \) that the implication \( A^+ \in F_{D^a}^* \Rightarrow A \in \Delta^+ \) holds. Suppose that \( A^+ \in F_{D^a}^* \)

\[
\Rightarrow \quad \exists \quad \text{a proof tree for } A^+ \text{ with respect to } D^a, \text{ whose depth shall be denoted } d.
\]
Correctness of Propagation Rules - 6

**d=0**

In this case, \( A^+ \) refers to an extensional relation and hence \( A \in \Delta^+ \) holds.

**d>0**

We assume that the implication holds for all delta facts \( A^+ \) having a proof tree wrt to \( D^a \) of depth less than \( d \). As \( d > 0 \), \( A^+ \) has children \( L^+_{1\sigma}, \text{new } L_{1\sigma},...,\text{new } L_{i-1\sigma}, \text{new } L_{i+1\sigma},...,\text{new } L_{n\sigma}, \text{old not } B_\sigma \) and \( A^+ \equiv B^+\sigma \) is derived via the propagation rule

\[
B^+ \leftarrow L^+_{1\sigma}, \text{new } L_{1\sigma}, ..., \text{new } L_{i-1\sigma}, \text{new } L_{i+1\sigma}, ..., \text{new } L_{n\sigma}, \text{old not } B.
\]

The child \( L^+_{i\sigma} \) of \( A^+ \) has a proof tree with respect to \( D^a \) of depth \(< d \).

- If \( L^+_{i\sigma} \) is a positive delta literal, then \( L^+_{i\sigma} \in F^*_D \) and from the induction hypothesis follows that \( L_{i\sigma} \in \Delta^+ \).

- If \( L^+_{i\sigma} \equiv \neg C^-_{i\sigma} \) is a negative literal, then \( C^-_{i\sigma} \in F^*_D \) and it can be shown by induction on the depth of proof trees with respect to \( D^a \) that then \( C_{i\sigma} \in \Delta^- \).

This shows that \( L_{i\sigma} \in D_{\text{new}} \). As the side literals new \( L_{1\sigma},...,\text{new } L_{n\sigma} \) are correctly evaluated it additionally follows that \( B_\sigma \in F^*_{\text{new}} \) due to \( B \leftarrow L_{1\sigma},..., L_{n\sigma} \in R^\text{new} \). The effectiveness test proves that \( B_\sigma \not\in F^*_{\text{old}} \) which finally shows that \( A \equiv B_\sigma \in \Delta^+ \).
Update Granularities

Let $D$ be a database and $\delta$ a transition from $D^{\text{old}}$ to $D^{\text{new}}$. Then $\delta$ induces a transition $\Delta$ from $(F^{\text{old}})^*$ to $(F^{\text{new}})^*$ which is a pair $\langle \Delta^+, \Delta^- \rangle$ of sets of ground atoms (delta sets).

**True Updates $\Delta_t$:**

\[
\Delta_t^+ = (F^{\text{new}})^* \setminus (F^{\text{old}})^* \\
\Delta_t^- = (F^{\text{old}})^* \setminus (F^{\text{new}})^*
\]

exact differences between the old and new states

**Safe Updates $\Delta_s$:**

\[
\Delta_t^+ \subseteq \Delta_s^+ \subseteq (F^{\text{new}})^* \\
\Delta_t^- \subseteq \Delta_s^- \subseteq (H_D \setminus F^{\text{new}})^*
\]

may additionally contain redundant insertions of facts $\in F^{\text{old}}$ as well as deletions of facts $\notin F^{\text{old}}$

**Potential Updates $\Delta_p$:**

\[
\Delta_t^+ \subseteq \Delta_p^+ \subseteq H_D \\
\Delta_t^- \subseteq \Delta_p^- \subseteq H_D
\]

may overestimate true insertions and deletions by redundant and even false ones
Update Granularities - Properties

Safe and potential updates are a useful and efficiently calculable approximation of the true changes. The most essential properties are as follows:

**True Updates** $\Delta_t$:

\[
\Delta_t^+ \cap \Delta_t^- = \emptyset
\]

\[
(F_{\text{new}})^* = (Fold * \setminus \Delta_t^-) \cup \Delta_t^+
\]

\[
(Fold)^* = (F_{\text{new}} * \setminus \Delta_t^+) \cup \Delta_t^-
\]

New and old database state can be constructed from the other one and $\Delta_t$.

**Safe Updates** $\Delta_s$:

\[
\Delta_s^+ \cap \Delta_s^- = \emptyset
\]

\[
(F_{\text{new}})^* = (Fold * \setminus \Delta_s^-) \cup \Delta_s^+
\]

\[
(Fold)^* \subseteq F_{\text{new}} * \cup \Delta_s^-
\]

The new state can be constructed from the old one and $\Delta_s$ but not the old one.

**Potential Updates** $\Delta_p$:

\[
\Delta_p^+ \cap \Delta_p^- = \emptyset
\]

\[
(F_{\text{new}})^* \subseteq Fold * \cup \Delta_p^+
\]

\[
(Fold)^* \subseteq F_{\text{new}} * \cup \Delta_p^-
\]

Neither the new nor the old state can be constructed from the other and $\Delta_p$. 
Propagation Rules for Safe Updates

In order to derive propagation rules for safe updates we start again from a stratifiable rule set $R$.

For each rule $A \leftarrow L_1, \ldots, L_n \in R$ and each body literal $L_i$ ($i=1,\ldots,n$) two propagation rules of the form

$$
A^+ \leftarrow L_i^{+},\text{new}\ (L_1,\ldots,L_{i-1},L_{i+1},\ldots,L_n).
$$

$$
A^- \leftarrow L_i^{-},\text{old}\ \text{Side},\text{new}\ \text{not A}.
$$

are in $R^\Delta$ where $\text{Side} \subseteq \{L_1,\ldots,L_{i-1},L_{i+1},\ldots,L_n\}$ is a subset of body literals such that the rule for $A^-$ remains to be safe.
Rules for Safe Updates – Example

\[ R \]

\[ R^\Delta s \]

\[
\begin{align*}
  p^+(X,Y) &\leftarrow c^+(X,Y), \text{new not } b(X,Y). \\
  p^+(X,Y) &\leftarrow b^-(X,Y), \text{new } c(X,Y). \\
  p^-(X,Y) &\leftarrow c^-(X,Y), \\
  p^-(X,Y) &\leftarrow b^+(X,Y), \\
  p^+(X,Y) &\leftarrow p^+(X,Z), \text{new } p(Z,Y). \\
  p^+(X,Y) &\leftarrow p^+(Z,Y), \text{new } p(X,Z). \\
  p^-(X,Y) &\leftarrow p^-(X,Z), \text{old } p(Z,Y), \text{new not } p(X,Y). \\
  p^-(X,Y) &\leftarrow p^-(Z,Y), \text{old } p(X,Z), \text{new not } p(X,Y). \\
\end{align*}
\]

Delta relation \( p^- \) may already provide redundant updates

Side literals needed to ensure safeness.
Comparison between the propagation of true updates ...

\[ R^\Delta \]

- \( p^+(X,Y) \leftarrow c^+(X,Y), \overset{\text{not}}{b^\text{new}(X,Y)}, \overset{\text{not}}{p^\text{old}(X,Y)}. \)
- \( p^+(X,Y) \leftarrow b^-(X,Y), \overset{\text{not}}{c^\text{new}(X,Y)}, \overset{\text{not}}{p^\text{old}(X,Y)}. \)
- \( p^-(X,Y) \leftarrow c^-(X,Y), \overset{\text{not}}{b^\text{old}(X,Y)}, \overset{\text{not}}{p^\text{new}(X,Y)}. \)
- \( p^-(X,Y) \leftarrow b^+(X,Y), \overset{\text{not}}{c^\text{old}(X,Y)}, \overset{\text{not}}{p^\text{new}(X,Y)}. \)

\[ R^\Delta_s \]

- \( p^+(X,Y) \leftarrow p^+(X,Z), \overset{\text{not}}{p^\text{new}(Z,Y)}, \overset{\text{not}}{p^\text{old}(X,Y)}. \)
- \( p^+(X,Y) \leftarrow p^+(Z,Y), \overset{\text{not}}{p^\text{new}(X,Z)}, \overset{\text{not}}{p^\text{old}(X,Y)}. \)

Delta relation \( p^- \) may already provide redundant updates
Propagation Rules for Potential Updates - 1

- For propagating potential updates, the truth value of the updated facts neither in the old nor in the new state is essential.

- Many approaches to UP have been introduced completely refraining from the evaluation of side literals and effectiveness tests:

  \[ p^+(X,Y) \leftarrow p^+(X,Z), p^{new}(Z,Y), \text{not } p^{old}(X,Y). \]

- For repairing unsafe rules, we encode non-ground facts by adorning the predicate symbols and dropping all unbound variables:

  \[ p^+_{bf}(X) \leftarrow p^+_bb(X,Z), \text{ Unsafe?} \]
The derivation of a fact, say $p_{bf}^+(c)$ indicates that all ground facts $p(c,c')$ of $H_D$ are considered potential insertions.

The proposed encoding is not faithful (cf. [Bry90]) since the distinct non-ground facts like $p^+(x,y,a)$ and $p^+(z,z,a)$ are equally represented by $p_{ffb}^+(a)$.

Adornment $a$: $p$ an $n$-ary predicate $\Rightarrow a=a_1,a_2,...,a_n$ with $a_i=\text{“b”}\lor \text{“f”}$

Encoded Literal $\varepsilon_a(L)$: $L$ is a literal, $a$ an adornment

- If $L$ is a base literal, then $\varepsilon_a(L):=L$.
- If $L=p(t_1,...,t_n)$ is a positive derived literal, then $\varepsilon_a(L):=p_a(L^b)$. where $L^b$ is the sequence of arguments of $L$ indicated as “$b$” in $a$.
- If $L=\text{not } A$ is a negative derived literal, then $\varepsilon_a(\text{not } L):=\text{not } \varepsilon_a(L)$. 

For each rule $A \leftarrow L_1, \ldots, L_n \in R$, each body literal $L_i$ ($i=1,\ldots,n$) and each adornment $a$ of $L$, two propagation rules of the form

$$
\begin{align*}
\varepsilon_b(A)^+ & \leftarrow \varepsilon_a(L_i)^+, \text{ new Side}^+ \\
\varepsilon_b(A)^- & \leftarrow \varepsilon_a(L_i)^-, \text{ old Side}^-
\end{align*}
$$

are in $R^\Delta$ where for $\pi \in \{+,-\}$, $\text{Side}^\pi \subseteq \{L_1, \ldots, L_{i-1}, L_{i+1}, \ldots, L_n\}$ is a subset of body literals such that $(L_i^\pi, \text{Side}^\pi)$ is safe, and $b$ is the adornment of $A$ which includes a “b” for each constant of $A$ and each variable of $A$ that occurs in $(L_i^\pi, \text{Side}^\pi)$. Any other position of $b$ is equal to “f”.

### Rules for Potential Updates – Example

**R**

\[ p(X,Y) \leftarrow c(X,Y), \text{ not } b(X,Y). \]
\[ p(X,Y) \leftarrow p(X,Z), p(Z,Y). \]

**\( R^{\Delta p} \)**

\[
\begin{align*}
p^{+}_{bb}(X,Y) & \leftarrow c^{+}(X,Y). \\
p^{+}_{bb}(X,Y) & \leftarrow b^{-}(X,Y). \\
p^{-}_{bb}(X,Y) & \leftarrow c^{-}(X,Y). \\
p^{-}_{bb}(X,Y) & \leftarrow b^{+}(X,Y). \\
p^{+}_{bf}(X) & \leftarrow p^{+}_{bb}(X,Z). \\
p^{+}_{bf}(X) & \leftarrow p^{+}(X),. \\
p^{+}_{ff} & \leftarrow p^{+}_{fb}(Z). \\
p^{+}_{ff} & \leftarrow p^{+}. \\
p^{+}_{fb}(Y) & \leftarrow p^{+}_{bb}(Z,Y). \\
p^{+}_{fb}(Y) & \leftarrow p^{+}_{bb}. \\
p^{+}_{ff} & \leftarrow p^{+}_{fb}(Y). \\
p^{+}_{ff} & \leftarrow p^{+}_{ff}. \\
p^{-}_{bf}(X) & \leftarrow p^{-}_{bb}(X,Z). \\
p^{-}_{bf}(X) & \leftarrow p^{-}_{bb}(X). \\
p^{-}_{ff} & \leftarrow p^{-}_{fb}(Z). \\
p^{-}_{ff} & \leftarrow p^{-}. \\
p^{-}_{fb}(Y) & \leftarrow p^{-}_{bb}(Z,Y). \\
p^{-}_{fb}(Y) & \leftarrow p^{-}_{bb}. \\
p^{-}_{ff} & \leftarrow p^{-}_{fb}(Y). \\
p^{-}_{ff} & \leftarrow p^{-}_{ff}. \\
\end{align*}
\]

redundant
Propagation of Potential Updates

\[ \text{However, if } p^+_ff \text{ can be derived, all ground facts } p(c,c') \text{ are represented and hence all other delta relations of } p \text{ are subsumed.} \]
Propagating Potential Updates – Simplified (1)

The particular rules of the example, however, always ultimately lead to the derivation of \( p^+_{ff} \) if there are any changes in \( c \) or \( b \).

Further simplification of potential update propagation.
A slightly modified example (unchanged semantics) shows how potential updates can be used for indicating relevant consequences for p due to changes in c or b.

Accumulating all nodes with an outgoing edge possibly affected by the base changes
Combining Different Propagation Rules

- Obviously, there is a **trade-off** between the **granularity** of induced updates, their **number** and the **complexity** of their evaluation.

- It would be useful if for **each** relation could be **separately decided** whether true, safe or potential updates are to be computed.

- This **cannot** be achieved by simply **mixing** propagation rules of different update classes.

**Main Idea:** If in the course of propagating true/safe updates a weaker class is referenced, the updates computed for the body literal have first to be restricted to those meeting the requirements of the higher class!
Combining Different Propagation Rules

Let $A \leftarrow W$ be the rule to be transformed, $L \in W$ the currently considered body literal, and $W' \equiv W \setminus \{L\}$ the side literals of $L$.

**True Updates**

Safe body updates have to pass the trueness test (old $\{\text{not } L | L\}$) before they can be further propagated:

- $A^+ \leftarrow L^+, \quad L^+, \quad \text{new } W', \quad \text{old not } A$.
- $A^- \leftarrow L^-, \quad \text{old } L, \quad \text{old } W', \quad \text{new not } A$.

old $\text{not } L$ not needed as it is subsumed by old $\text{not } A$. 

[Diagram showing the propagation rules with highlighted parts]
Combining Different Propagation Rules

Let $A \leftarrow W$ be the rule to be transformed, $L \in W$ the currently considered body literal, and $W' = W \setminus \{L\}$ the side literals of $L$.

True Updates

Potential body updates have to pass the safeness (new $\{\text{not } L \mid L\}$) and the trueness (old $\{\text{not } L \mid L\}$) tests. Furthermore it is necessary to generate propagation rules for each possible adornment $a$ of $L$:

\[
\begin{align*}
A^+ & \leftarrow \varepsilon_a(L^+) , \quad \text{new } L \text{, new } W' , \quad \text{old not } A . \\
A^- & \leftarrow \varepsilon_a(L^-) , \quad \text{old } L , \quad \text{old } W' , \quad \text{new not } A .
\end{align*}
\]
Combining Different Propagation Rules

Let $A \leftarrow W$ be the rule to be transformed, $L \in W$ the currently considered body literal, and $W' = W \setminus \{L\}$ the side literals of $L$.

**Safe Updates**

Potential body updates have to be checked for safeness (new $\{\text{not } L \mid L\}$) on the new state. Furthermore, propagation rules have to be generated for each possible adornment $a$ of $L$:

\[
\begin{align*}
A^+ & \leftarrow \varepsilon_a(L^+), \\
A^- & \leftarrow \varepsilon_a(L^-),
\end{align*}
\]

new $L$, new $W'$, old $Side$, new $\text{not } A$. 